APPENDIX III
Derivation and Calculation of the Energy Required to Reduce the Rate of Rotation of Mars

Starting with the equation for kinetic energy,

$$KE = \frac{1}{2} m \cdot v^2 \qquad (28)$$

where:
KE = Kinetic Energy
m = mass
v = velocity

For a rotating mass, Equation-28 can be written as,

$$KE = \frac{1}{2} (\rho \cdot vol) \cdot (\omega \cdot r)^2 \qquad (29)$$

where:
ρ = density
vol = volume
ω = angular velocity
r = radius of rotation of mass

Equation-29 can be re-written as,

$$KE = \frac{\omega^2 \cdot \rho}{2} vol \cdot r^2 \qquad (30)$$

By analyzing a differential volume, as shown in Figure-9 Equation-30 can be expressed as,

$$\delta KE = \frac{\omega^2 \cdot \rho}{2} \delta vol \cdot r^2 \qquad (31)$$

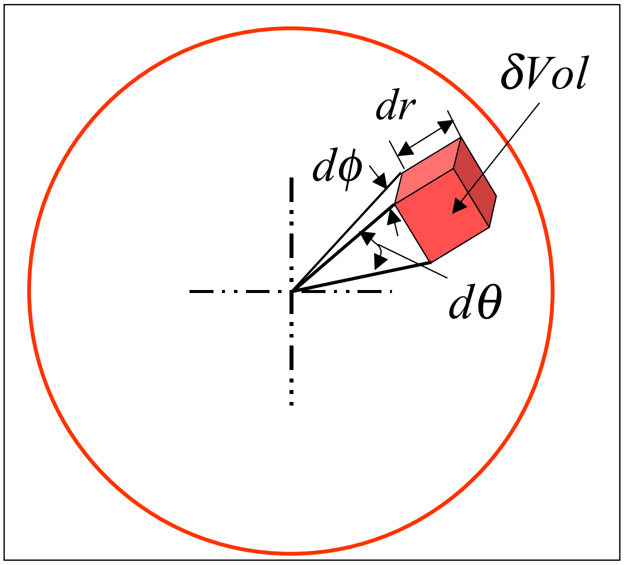

Figure-9 Differential volume in a rotating sphere used in the calculation of the Mars lithosphere energy of rotation.

The dimensions of the differential volume can be substituted into Equation-31 to give,

$$KE = \frac{\omega^2 \cdot \rho}{2}\left(r \cdot \delta\theta \cdot r \cdot \delta\phi \cdot \delta r\right) \cdot r^2 \qquad (32)$$

which can be simplified to,

$$KE = \frac{\omega^2 \cdot \rho}{2}\left(r^4 \cdot \delta\theta \cdot \delta\phi \cdot \delta r\right) \qquad (33)$$

The kinetic energy of the rotating lithosphere can now be calculated by integrating Equation-33 over the volume of the lithosphere:

$$KE = \frac{\omega^2 \cdot \rho}{2}\int_{0\deg}^{360\deg}\int_{90\deg}^{270\deg}\int_{r_1}^{R_{Mars}} r^4 dr \cdot d\theta \cdot d\phi \qquad (34)$$

where:
r_1 = the inner radius of the lithosphere
R_{Mars} = the outer radius of Mars

The energy required to reduce the rotation from ω_1 to ω_2 is therefore,

$$E_{spin} = \frac{\omega_1^2 \cdot \rho}{2}\int_{0\deg}^{360\deg}\int_{90\deg}^{270\deg}\int_{r_1}^{R_{Mars}} r^4 dr \cdot d\theta \cdot d\phi - \frac{\omega_2^2 \cdot \rho}{2}\int_{0\deg}^{360\deg}\int_{90\deg}^{270\deg}\int_{r_1}^{R_{Mars}} r^4 dr \cdot d\theta \cdot d\phi \qquad (35)$$

Equation-35 can be simplified to:

$$E_{spin} = \frac{\left(\omega_1^2 - \omega_2^2\right)\cdot \rho}{2} \int_{0\,\deg}^{360\,\deg} \int_{90\,\deg}^{270\,\deg} \int_{r_1}^{R_{Mars}} r^4 dr \cdot d\theta \cdot d\phi \qquad (36)$$

where:
ω_1 = is the original spin rate
ω_2= is the final spin rate

By using Equation-36 it can be seen that it would take between $4.42\text{x}10^{25}$ and $9.56\text{x}10^{25}$ Joules (depending on the thickness of lithosphere assumed, 110 – 260 km) to reduce the Mars spin rate by 20% (19.5 hr day to the current 24.5 hr. day). This is only 8% to 18% (depending on the thickness of lithosphere assumed) of the total energy of impact of the Hellas Basin. In other words, there was sufficient amount of energy in the Hellas impact to slow the rate of rotation down by 20%.