## APPENDIX II
## Derivation and Calculation of the Energy of Rupture for Mars

For analysis purposes, Mars will be treated as a spherical pressure vessel. When performing structural analysis of pressure vessels, the equations for thin-walled pressure vessels should be used when the ratio of the radius to the wall thickness is greater than 10 [14].

Willemann and Turcotte [15] show that the Mars lithosphere thickness is estimated to be anywhere from 110 km to 260 km. Therefore, the ratio of the radius of Mars (3375 km) to the lithosphere thickness may range from 30 to 13 and the equations for thin-walled pressure vessels apply.

The energy required to rupture the shell of a thin-walled pressure vessel will be derived in the following section. This equation will then be applied to Mars to determine the energy required to rupture the lithosphere.

A spherical pressure vessel experiences equal tension in both the axial and circumferential directions when exposed to an internal pressure. The circumferential stress will be referred to as hoop stress ($\sigma_H$) and the axial stress will be referred to as longitudinal stress ($\sigma_L$). This is shown in Figure-2.

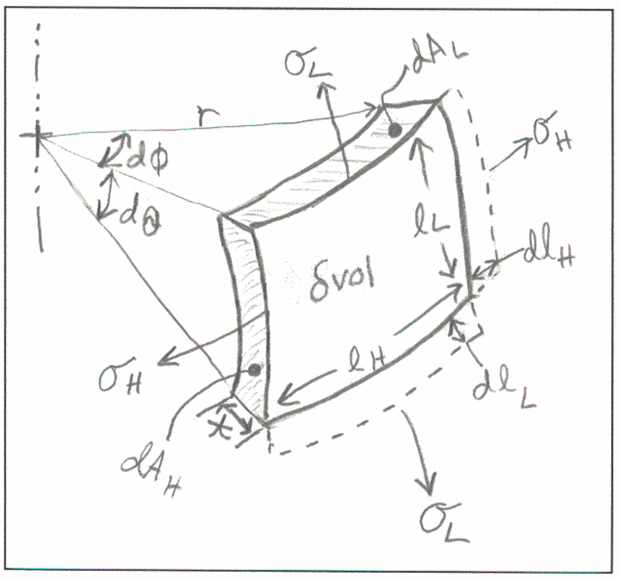


**Figure-2**. Differential strain on a differential volume of the Mars lithosphere.

Work is defined as:

$$W = F \cdot X \tag{5}$$

where:
$F$ = Force
$X$ = Distance

The external work done on an elastic member in deforming it is transformed into strain energy [16]. In other words,

$$WorkDone = \Delta Energy \tag{6}$$

$$dW = dE = T \cdot dc \tag{7}$$

where:
$dc$ = differential change in circumference (substitute for $X$ in Equation-5)

$$T = Tension$$

(substitute for $F$ in Equation-5)

knowing that,

$$T = \sigma \cdot Area \qquad (8)$$

equation-7 can be written as:

$$dE = \sigma \cdot A \cdot dc \qquad (9)$$

Because the stress in a spherical pressure vessel experiences both hoop stress and longitudinal stress, the total energy of rupture is equal to the sum of the energy of rupture due to hoop stress and the energy of rupture due to longitudinal stress,

$$dE = dE_H + dE_L \qquad (10)$$

Since the differential strain = $d\varepsilon = dc/l$, the differential energy required to strain a differential "hoop section" by $d\varepsilon_H$ is defined as:

$$dE_H = (\sigma_H \cdot dA_H)(l_H \cdot d\varepsilon_H) \qquad (11)$$

where:
$dA_H$ = the differential area in the hoop direction
$l_H$ = the total length along the hoop direction
$d\varepsilon_H$ = the differential strain in the hoop direction

Similarly the differential energy required to strain a differential "longitudinal section" by $d\varepsilon_L$ is defined as:

$$dE_L = (\sigma_L \cdot dA_L)(l_L \cdot d\varepsilon_L) \qquad (12)$$

where:
$dA_L$ = the differential area in the longitudinal direction
$l_L$ = the total length along the longitudinal direction
$d\varepsilon_H$ = the differential strain in the longitudinal direction

The following relations can now be substituted into equations 11 and 12:

$$dA_H = t \cdot r \cdot d\theta \qquad (13)$$
$$dA_L = t \cdot r \cdot d\phi \qquad (14)$$
$$l_H = r \cdot d\phi \qquad (15)$$
$$l_L = r \cdot d\theta \qquad (16)$$
$\sigma = \sigma_H = \sigma_L$ (assuming an isotropic sphere) (17)
$d\varepsilon = d\varepsilon_L = d\varepsilon_H$ (since longitudinal stress = hoop stress) (18)

We now have the following:

$$dE_H = \sigma \cdot t \cdot r^2 \cdot d\theta \cdot d\phi \cdot d\varepsilon \qquad (19)$$

and,

$$dE_L = \sigma \cdot t \cdot r^2 \cdot d\theta \cdot d\phi \cdot d\varepsilon \qquad (20)$$

Since the hoop stress is equal to the longitudinal stress,

$$dE = 2\sigma \cdot t \cdot r^2 \cdot d\theta \cdot d\phi \cdot d\varepsilon \qquad (21)$$

$d\varepsilon$ is defined as:

$$d\varepsilon = \frac{d\sigma}{E_{\mathrm{mod}}} \qquad (22)$$

where:
$E_{mod}$ = modulus of elasticity
$\sigma$ = the tensile stress.

Which gives the following,

$$dE = \frac{2 \cdot \sigma \cdot t \cdot r^2}{E_{\mathrm{mod}}} \cdot d\theta \cdot d\phi \cdot d\sigma \qquad (23)$$

The energy of rupture required to rupture the Martian lithosphere can be determined by integrating Equation-23 from zero to ultimate stress throughout the surface area of Mars.

$$E = \int_{0}^{\sigma_{ULT}} \int_{0}^{360} \int_{-90\,\mathrm{deg}}^{90\,\mathrm{deg}} \frac{2 \cdot \sigma \cdot t \cdot r^2}{E_{\mathrm{mod}}} \cdot d\theta \cdot d\phi \cdot d\sigma \qquad (24)$$

The following values can now be substituted into equation (24).

- $r$ = Mars radius = 3375 km
- $t$ =Lithosphere thickness = 110-260 km [14]
- $E_{mod}$ = 6.5x$10^{10}$ Pa (9.427x$10^{6}$ psi) [17]
- $\sigma_{ULT}$ = 5.0x$10^{8}$ Pa (7.25x$10^{4}$ psi) [17]

The lithosphere depth is used instead of the Valles Marineris depth of 7 km for the value of "*t*" because the strain energy was applied throughout the thickness of the crust, which in-turn resulted in one of two scenarios:

1. The lithosphere ruptured throughout its thickness resulting in magma to swell up and fill in the gap in the crust until it was level with the surrounding lowlands. Thus explaining why the Valles Marineris is the same depth as the surrounding lowlands.
2. The lithosphere only ruptured along its surface to a depth of 7 kilometers. Since the outer layers of the crust lie at greater diameters, for each incremental increase in diameter due to the internal pressure, the outer crustal layers must strain at a greater rate than the internal layers.

Using equation-24, and depending on the value of lithosphere thickness used (110 to 260 km), the energy required to rupture the Martian lithosphere is calculated to be 1.90x$10^{26}$ to 4.50x$10^{26}$ Joules, which is 36% to 84% of the total energy input to the Martian system by the Hellas impactor.