Appendix
Rotational components in a disturbed gyroscope

The rotational components in a disturbed gyroscope are connected to each other by the following equation, due to Laplace, which expresses the principle of conservation of energy:

1) $\boldsymbol{J_o}\, W^2 = \boldsymbol{J_o}\, w^2 + \boldsymbol{J_p}\, w_p^2 = \boldsymbol{J_i}\, w_i^{\,2}$

where: W = speed of rotation of the undisturbed gyroscope
w = speed of rotation of the gyroscope around its main axis
w_p = speed of precession
w_I = speed of instantaneous rotation
J_o = main momentum of inertia
J_p = momentum of inertia related to the precession axis
J_i = momentum of inertia related to the axis of instantaneous rotation

The value of the torque developed by a disturbing force F_p, applied to the main axis of the gyroscope with an angle β, is evidently given by:

2) $C_p = R\ F_p\ \mathrm{sen}\beta$

where R is the arm of the force, that is the distance of his point of application from the centre of the gyroscope.

Instant by instant the gyroscope precedes around an equatorial axis, but the resulting motion of he main axis describes a cone, with the axis parallel to the force, an opening angle of 2 β and its vertex at the centre of the gyroscope. The main axis, therefore, appears to rotate with angular speed ω_{pa} around an axis parallel to the disturbing force.

The value of ω_{pa} is given by the following equation:

3) $$w_{pa} = \frac{w_p}{senb}$$

Equations 1), 2) and 3) allow us to study exhaustively the behaviour of a disturbed gyroscope, by means of an essentially graphic method.

Given a gyroscope let's draw, on the basis of its inertia ellipse, another ellipse whose semi-axis are respectively:

$$a = \sqrt{\frac{J_o}{J_p}}\ ; \qquad b = \sqrt{\frac{J_o}{J_o}} = 1$$

Every radius of the ellipse, *r(q)* , where: θ = 0 ÷ 2π, would obviously have the value:

$$r_q = \sqrt{J_o / J_q}$$

where J_q is the momentum of inertia of an axis forming an angle *q* with the main axis.

If we put $\Omega^2 = 1$, for equation 1) every radius *r(q)* is proportional to the speed of rotation that the gyroscope has to have around axis *q* to keep its initial energy unchanged.

The end of the arrows representing Ω and ω_I, therefore, always fall on the ellipse, while all the other rotational components have to be found inside the ellipse. This ellipse allows us to analyse exhaustively the behaviour of all the rotational components of the gyroscope, bound as they are by equation 1) (see fig.1).

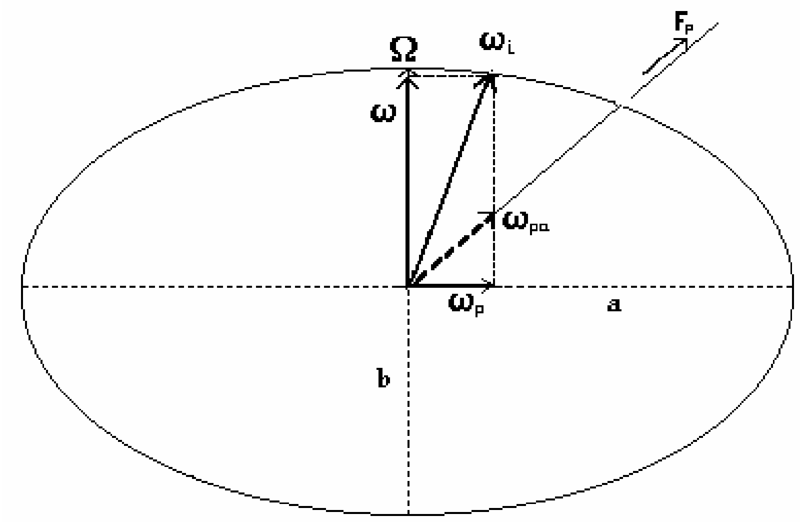


fig.1

The meaning of the rotational components shown in fig. 1 is easily understood. A gyroscope subjected to a disturbing torque reacts generating an exactly equal and opposed torque. This is achieved by means of a precession movement, ω_p, around an equatorial axis, which makes the gyroscope rotate "unbalanced", that is rotate instant by instant around an axis, which forms with the main axis an angle β proportional to the disturbing torque. The instantaneous rotation, ω_I, is given by the sum of the rotation around the main axis, ω, plus the rotation of precession, ω_p.

When a gyroscope is subjected to a disturbing force F_p, of increasing value, ω_p grows and as a consequence ω_I moves towards ω_{pa}.

When F_p reaches a certain value F_{pa} (see calculations further on), we will have:

$\omega_I = \omega_{pa}$

At that precise moment the axis of instantaneous rotation coincides with the axis of apparent precession, and becomes fixed with respect to both, the space and the gyroscope. This is a very special condition in which the system composed by the gyroscope and the disturbing torque behaves like a non-disturbed gyroscope, with only a single rotational component, Ω' (see fig. 2). This axis, therefore, becomes the new axis of rotation of the system.

If at this point force F_p diminishes again, the system behaves like a gyroscope to which is applied a torque of value:

$C'_p = C_{pa} - C_p$

Therefore the new axis of rotation begins to precede around the main axis, moving on the surface of a cone. As a consequence ω_I' moves back towards the main axis, following the same path it has run along in the previous journey. Value and direction of the gyroscope's rotational components in this case are represented in fig.2

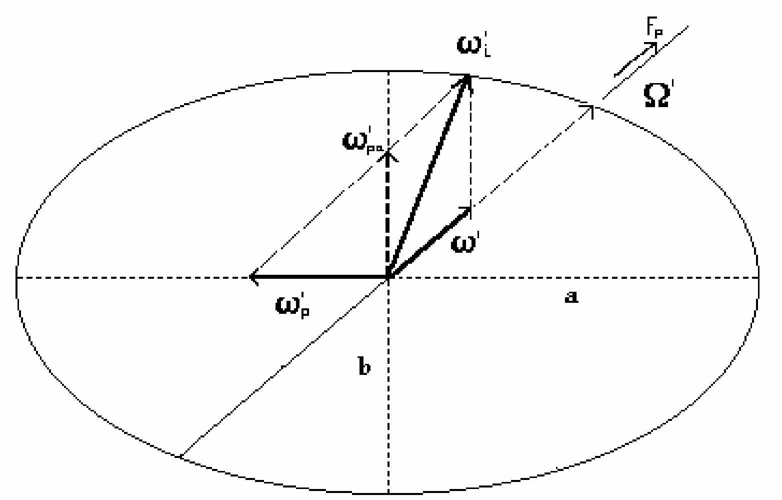


fig.2

Due to the principle of conservation of energy we will evidently have:

$$J_{pa}\, W'^2 = J_o\, w'^2 + J'_p w_p'^2 = J_i\, w_i'^2 = J_o\, W^2$$

For each value of the disturbing force, F_p, the speed of the instantaneous rotation is exactly the same both ways, there and back, that is $\omega_i' = \omega_i$. The other rotational components, instead, change considerably and ω'_p has direction opposite to that of ω_p . This is justified by the fact that while F_p is growing, the main axis rotates around axis $\omega_{pa.}$ In the "return journey " the contrary happens: it is the axis of ω' (now fixed in respect to the body of the gyroscope) that rotates around the main axis.

The most important fact is that along the ω' axis we have a rotational component which is fixed in respect to the gyroscope. This means that the gyroscope keeps "memory" of the position of the new axis of rotation. That rotational component, therefore the "memory", is cancelled only if and when F_p is completely zeroed. If F_p should not be zeroed , the gyroscope would keep this rotational component, and therefore the "memory", indefinitely.

Behaviour of a semifluid gyroscope like the Earth

The behaviour of the Earth as a gyroscope is subject to some peculiarities due to the fact that the planet is not a homogenous and rigid solid, but is made up of liquid parts inside and outside an intermediate plastic shell.

Suppose the planet is hit by large celestial bodies at high speed. The impact develops an impulsive torque, that according to the size and speed of the impacting mass can have a very high peak value, as high as the highest reaction torque possibly developed by Earth.

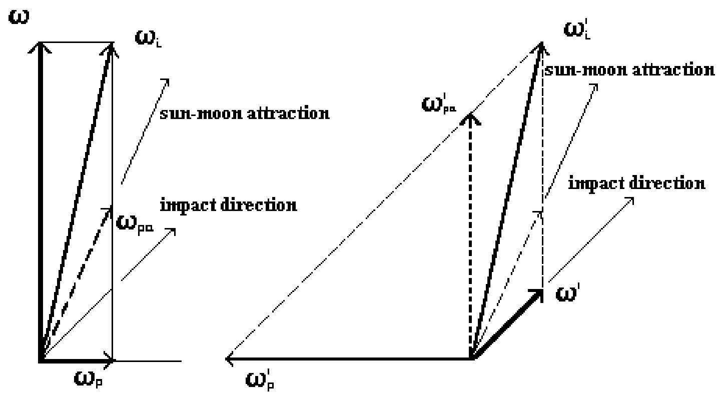
Graphics of fig.1 and fig.2, can help us to understand what happens in this case.

As soon as the torque developed by the impact starts growing, the ω_i moves in the direction of ω_{pa}, parallel to the direction of impact. If the impact develops a torque of sufficient value, ω_i will coincides with ω_{pa}. On that instant the axis of ω_{pa} becomes axis of permanent rotation. As soon as the torque value decreases, the axis of ω_i returns quickly towards the main axis, but following a different path as shown in fig. 2. As soon as the shock ceases, an instant later, the Earth should again return to rotate around its natural axis, without any further repercussion. But it is not necessarily so.

To cancel the "memory" of the new axis of rotation, and have the gyroscope rotating again around the main axis, it is necessary that the torque be completely spent. Unfortunately, there are good probabilities that this may not happen. We know that Earth is permanently subjected to a torque generated by the gravitational forces of the sun and the moon on the equatorial bulge. This torque is millions of times smaller than the one developed by the impact, but its role is fundamental.

If at that moment it has a different direction than the one developed by the impact itself, as soon as the shock is exhausted, the Earth instantly recovers its previous axis of rotation and all ends there. If, instead, the torque due to the Sun-Moon attraction has the same direction of the torque caused by the celestial body, it is added to this, and contributes in its small way to the instantaneous change of the position of the poles. A few instants later the shock exhausts itself while the Sun-Moon gravitational attraction continues, and however small, it nonetheless develops a torque higher than zero. Therefore the "memory" of the axis around which the Earth has rotated during the impact, even for a very short moment, cannot be cancelled.

In this case the Earth actually behaves like a gyroscope whose main axis coincides with the one adopted during the impact, subjected to a disturbing torque equal but opposite to the torque generated by the impact. The overall motion is apparently exactly the same, but in reality there are fundamental differences, as illustrated in fig.3 .



a **b**

fig.3

Graphics n. 3.a and 3.b represent the situation of Earth's rotational components immediately before (3.a) and after (3.b) the impact, in the case in which the Sun-Moon disturbing force has the same direction of the force developed by the impact. (To make it easier to represent them, the precession rotational components in the illustration are greatly exaggerated; in reality they are more than one million times smaller than the main rotation. The rationale however does not change).

Apparently the situation has not changed, because ω_i is exactly equal to ω'_i, and ω' has the same magnitude as the previous precession speed ω_{pa}. There is however a crucial difference: at this point ω' is the only rotational component "fixed" with respect to the Earth's body. Thus, the axis of ω' has become axis of permanent rotation. The rotation around it is extremely small (one million of times smaller than the main rotation), but it develops in any case a centrifugal force strong enough to form an equatorial bulge (of a few meters) around its axis of rotation.

If the Earth was a solid gyroscope, this situation would last indefinitely unchanged. The planet, however, is covered by a thin layer of water, which reacts immediately to any change of motion.

Sea water begins to move towards the new equator, and as this happens the value of ω' increases again, therefore increasing the force which makes the water move towards the new equator, which in turns makes more water move towards the equator and so on. This process gradually accelerates, until the centrifugal force developed by ω' grows strong enough to induce deformations of the Earth's mantle.

From here on the equatorial bulge is quickly reformed around the new axis of rotation and Earth will soon be stable again, with a different axis of rotation and different poles.

Value of the reaction torque developed by Earth

The value of the reaction torque developed by a gyroscope, when rotating around an axis different from the main, can be calculated (see fig, 4) reckoning the torque developed by the element of mass, dm, rotating around the axis of ω_i:

$C_i = F_i\, b$

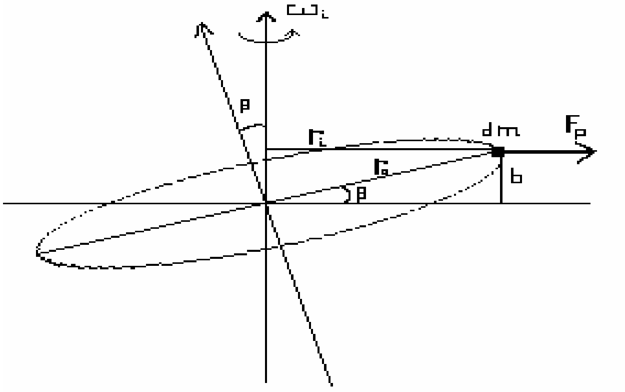


fig. 4

where:

$F_i = dm\, w_i^2\, r_i = dm\, w_i^2\, r_o \cos b$ is the centrifugal force;

$b = r_o\, \mathrm{sen} b$ is the arm of the torque.

We have therefore:

$C_i = dm\, r_o^2\, \omega_i^2\, \mathrm{sen}\beta \cos\beta = dJ_o\, \omega_i^2\, \mathrm{sen}\beta \cos\beta = ½\, dJ_o\, \omega_i^2\, \mathrm{sen}2\beta$

where $dJ_o = dm\, r_o^2$ is the momentum of inertia of mass dm with respect to the main axis.

For a ellipsoid of revolution we will have therefore:

4) $\boldsymbol{C = (J_o - J_p)\, w_i^2\, \mathrm{sen} b \cos b = ½\, J_r\, w_i^2\, \mathrm{sen}2b}$

where $J_r = (J_o - J_p)$ is the momentum of inertia of the bulge.

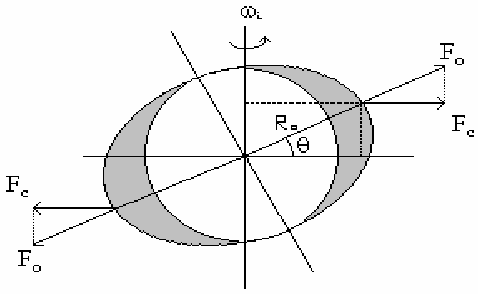


fig 5

Equation 4) shows that a gyroscope may develop a reaction torque only if $J_o \neq J_p$. In the case of it being perfectly spherical, it would rotate indifferently around whatever axis and it wouldn't have any stability.

This is due to the fact that in a rotating homogenous sphere, all centrifugal forces balance each other and there is no reaction torque, no matter what the axis of rotation is. Only the equatorial bulge can develop a reaction torque

Value of the stabilising torque developed by the equatorial bulge

From equation 4) we see that the maximum reaction torque possibly developed by a gyroscope is reached when $\beta = 45^o$:

$C_m = ½\, J_r\ \omega_i^2$

For Earth the value of ω_i is almost equal to that of the main rotation, so we can assume that:

$\omega_i^2 \cong (2\pi / 8,64)^2\, 10^{-10} = 5,28\ .\ 10^{-9}$ sec.$^{-2}$

The calculation of J_r can be made by using the value of the centrifugal force, $F_{o,}$ developed by the equatorial bulge due to the Earth's rotation, as calculated by Gallen and Deininger for Hapgood (see insert at the end):

F_o = 4,1192. 10^{19} kg.

For an approximate calculation we can put:

$J_r \cong M_r\, R_o^2$

$F_o \cong M_r\ \omega_i^2\, R_o = J_r\, \omega_i^2 / R_o$

where M_r is the mass of the bulge and R_o the radius of the Earth.

We have then:

$J_r \cong F_o\, R_o / \omega_i^2 \cong 5\ 10^{34}$ kgmt^2

And finally, thanks to equation 4) we have:

4') $\mathbf{C = ½\ J_r\, w_i^2\, sen2b = 1,38\ 10^{26}\, sen2b}$ kgmt

For $\beta = 45^o$ we have :

$C \cong 1,38\ 10^{26}$ kgmt

which is the maximum reaction torque possibly developed by Earth.

Calculation of the size an asteroid should have to trigger a shifting of the poles

According to equation 4) to overtake the reaction torque developed at an inclination, for instance, of 20^o, an asteroid hitting the Earth must develop an impulsive torque of the following value:

$C_{20°} = 8,87\ .\ 10^{25}$ Kgmt.

It is therefore easy to estimate the size and speed that such an asteroid must have.

The impulsive force F_i developed on impact with Earth by the asteroid is given by:

$F_i = M_a.a$

where:

$a = dv/dt$ is the acceleration the asteroid undergoes on impact

M_a is the mass of the asteroid

To calculate the acceleration, *a*, we can assume the asteroid has, on impact, a speed:

$v = 5\ .\ 10^4$ mt/sec.

To calculate dt we have to rely on an estimate. In a conservative way, considering the depth of known craters, we can presume that the depth of the crater caused by that impact to be 500 m, which means that the speed of the asteroid decreases from 5.10^4 to 0 mt/sec, in a space of 500 meters. The time in which this happens is approximately one hundredth of a second, that is:

$dt = 0{,}01$ sec.

The average acceleration of the asteroid will therefore be:

$a_m = dv/dt = 5.10^4 / 0,01 = 5.\ 10^6$ m/sec^2

The acceleration peak is certainly much higher. In a conservative calculation we can assume it to be double the average value. We will have then:

$a = 5.10^4 / 0.005 = 10^7$ mt/sec^2

And therefore:

$F_i = M_a . 10^7$ kg

The torque developed by this force will obviously be:

$C_i = F_i . R_i$

where R_i is the arm of the torque.

The value of R_i can be between 0 and $R_o \cong 6{,}4\ 10^6$ mt, that is the radius of the Earth. For statistical reasons we can put:

$R_i = ½\ R_o = 3{,}2\ 10^6$ mt

The mass of the asteroid will therefore be:

$$M_a = \frac{F_i}{a} = \frac{C_i}{R_i a} = \frac{8{,}87.10^{25}}{3{,}2.10^6.10^7}$$ = **2,77 10^12 kg**

If the density of the asteroid is of 3 Kg/dm^3, we will have a volume of:

$V_a = 0{,}92$ km^3

that is then a lithic asteroid of approximately a 1000 metres diameter. This calculation is very conservative. We can realistically suppose that an object of half that size is enough to develop a torque of sufficient value for a shift of the poles.

Gallen's calculation of the stabilising centrifugal effect of the equatorial bulge of the Earth

Let the equations of the sphere and the ellipsoid of revolution be:

1) $x^2 + y^2 + z^2 = b^2$

2) $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

where the axis of y is the axis of revolution. Take as the element of mass, dM, the ring generated by revolving the rectangle dxdy about the axis of y. We have:

3) $dM = 2\pi\delta x\ dxdy$

where δ is the density. For each particle of the ring the centrifugal acceleration is the same, being equal to $\omega^2 x$, where ω is the constant angular velocity in radiants per second.

The element of centrifugal force, dF, exerted by the ring is then:

4) $dF = \omega^2 x\ dM = 2\pi\delta\omega^2 x^2\ dxdy$

Integrating equation (4) with respect to x and y, there results:

5) $$F = 2pdw^2 \int_{-b}^{b} \int_{\sqrt{b^2-y^2}}^{\frac{a}{b}\sqrt{b^2-y^2}} x^2 dxdy = \frac{p^2 dw^2}{4} b\left(a^2 - b^2\right)$$

In equation (5) F is expressed in dynes when δ is given in grams per cubic centimeter, and a and b in centimeters. The quantity ω may be replaced by $2\pi n$, where n is revolutions per second. The Earth makes one complete revolution in 86,164.09 mean solar seconds.

Mrs. Deininger's computation based on Gallen's calculus

Computation of centrifugal force produced by rotation of the bulge,

A. Essential data:

1. The attached formula should apply to the bulge taken as 13.3443 miles at the equator, not the bulge as it would be if there were no flattening at the poles.
2. In making the calculation, Hapgood asked Mrs Harriest Deininger, of the Smith College faculty, to subtract three miles from the depth of the bulge, because he was concerned with a purely mechanical action of stabilisation, in which water could not have effect. (He later recognised that he subtracted about three miles too much, because he had disregarded isostasy, which in this case makes it probable that the rock under the oceans has a density higher than the density of the rock of the continents; so he should have subtracted the weight rather than the volume of the water. This however is a minor correction)
3. Mrs. Deininger actually took the depth of the bulge as nine miles, without the water.

B. Calculation:

$$F = \frac{p^2 s w^2}{4} b\left(a^2 - b^2\right)$$

where s = density in gm/cc
a = radius of Earth at bulge in cm
b = radius of Earth at poles in cm
w = 2 - n r = rps

2) $F = \pi^4 s n^2 . b(a^3 - b^3)$

where π = 3,1415
s = 2,7 gm/cm^3
n = 1/86.164
b = $6{,}402 . 10^8$ cm
a = $6{,}4165 . 10^8$ cm (using nine miles or 1,450,000 cm as depth of bulge)

3) ***F = $4{,}0368 . 10^{25}$ dine = $4{,}1192. 10^{19}$ kg.***

- Charles Hapgood, “The Path of the Pole”, Chilton Book Co, Philadefphia, 1970
- R. F. Deimel, “Mechanics of the Gyroscope. The dinamics of Rotation”, Dover Edition, 1950
- G.W. Wetherill, “ The Apollo objects”’Scientific American, May 79
- Tom Geherels, “Collision with Comets and Asteroids”’ Scientific American, March 96
- E. Spedicato, “Apollo objects, Atlantis and the deluge: a catastrophical scenario for the end of the last glaciation”, Quad. 90/22, 1990, Bergamo University, Italy